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Confining membranes and dimensional reduction

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Abstract: The dual theory describing the 4D Coulomb gas of point-like magnetically charged objects, which confines closed electric strings, is considered. The respective generalization of the theory of confining strings to confining membranes is further constructed. The same is done for the analogous SU(3)-inspired model. We then consider a combined model which confines both electric charges and closed strings. Such a model is a mixture of the above-mentioned Coulomb gas with the condensate of the dual Higgs field, where the latter one is described by the dual abelian Higgs model. It is demonstrated that in a certain limit of this dual abelian Higgs model, the system under study undergoes naively the dimensional reduction and becomes described by the (completely integrable) 2D sine-Gordon theory. In particular, at finite temperature, this fact leads to the phase transition of the Berezinskii-Kosterlitz-Thouless type with the respective critical temperature expressed in terms of the parameters of the dual abelian Higgs model. However, it is finally discussed that the dimensional reduction is rigorously valid only in the strong coupling limit of the original 4D Coulomb gas. In such a limit, this reduction transforms the combined model into the 2D free bosonic theory.

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Contents

1. Introduction

It is known that in 4D there exists a confining medium which, contrary to the 4D compact QED, possesses the property of confinement at arbitrary values of the electric coupling [1]. Such a medium is a gas of point-like magnetically charged objects with a finite action. The lattice expression for the part of the partition function describing magnetic objects reads [1]:

$$\mathcal{Z}_m = \sum_{\{q_x\}} \exp\left[-2\pi^2 g_m^2 \sum_{x,x'} q_x \Delta^{-1}_{xx'} q_{x'}\right]. \tag{1.1}$$

Here, Δ is the 4D lattice laplacian, q_x's are the magnetic fluxes produced by these objects through 3D hypercubes, and g_m is the magnetic coupling constant of dimensionality $(\text{mass})^{-1}$. This model can be viewed as an analogue of compact QED, where the rôle of the vector potential is played by the antisymmetric tensor field (the so-called Kalb-Ramond [2] field). Clearly, such a field can couple only to a surface, rather than to a line as the ordinary vector potential does. Owing to this fact, Wilson's criterion of confinement in the theory (1.1) should be formulated not for the usual Wilson loop, but for its analogue defined at some closed 2D surface. In the same way as the contour of the conventional Wilson loop is a trajectory of an electrically charged point-like particle, such a surface can naturally be identified with a world sheet of a closed electric string. Thus, confinement in the model (1.1) is the confinement of closed electric strings, and its criterion is the volume law for the above-mentioned analogue of the Wilson loop. Such a volume law is a natural generalization to the case of strings of the area law for the usual Wilson loops which correspond to point-like particles (see e.g. [3]).

In the present paper, we shall make these ideas quantitative for the continuum version of the model (1.1). Our first aim, which will be realized in section 2, will be the construction of a generalization of the theory of confining strings [4], which confine electric charges, to the case of membranes. The resulting theory of confining membranes will be a theory which confines closed electric strings in the sense that the world volume of a membrane is a 3D hypersurface bounded by the world sheet of an electric string. Such membranes will turn out to possess stiffness and are therefore of the same type as biomembranes, whose stiffness was for the first time emphasized in ref. [5]. Also, in section 2, we shall generalize the obtained theory of confining membranes to the SU(3)-inspired case. In section 3, we shall consider one concrete example of the model where closed electric strings exist, namely dual abelian Higgs model in the London limit. In particular, we shall explore there a possible dimensional reduction of a combined theory, which confines both electric strings and point-like charges, from 4D to 2D. Finally, the main results of the paper will be summarized in the Conclusions.

2. Confining membranes

The initial action of the model which describes the free Kalb-Ramond field and a plasma of point-like magnetic objects reads [1]

$$S = \frac{1}{12}\int d^4x \left(H_{\mu\nu\lambda} + H^m_{\mu\nu\lambda}\right)^2 . \tag{2.1}$$

Here, $H_{\mu\nu\lambda} = \partial_\mu h_{\nu\lambda} + \partial_\lambda h_{\mu\nu} + \partial_\nu h_{\lambda\mu}$ is the field strength tensor of the Kalb-Ramond field $h_{\mu\nu}$, $[h_{\mu\nu}] =$ (mass), satisfying the analogue of the Bianchi identity, $\varepsilon_{\mu\nu\lambda\rho}\partial_\mu H_{\nu\lambda\rho} = 0$. Next, $H^m_{\mu\nu\lambda}$ is the analogue of the monopole field strength tensor in compact QED, obeying the equation

$$\frac{1}{6}\varepsilon_{\mu\nu\lambda\rho}\partial_\mu H^m_{\nu\lambda\rho} = 2\pi g_m \rho_{\rm gas} . \tag{2.2}$$

Here, $\rho_{\rm gas}(x) = \sum\limits_{a=1}^{N} q_a \delta(x - z_a)$ is the density of point-like objects, carrying magnetic charges q_a's (in the units of g_m) and located at the points z_a's.

The dualization of the theory (2.1) with the subsequent summation over the grand canonical ensemble of magnetic objects with $q_a = \pm 1$ yields the action of the 4D sine-Gordon theory:

$$S_{\rm SG} = \int d^4x \left[\frac{1}{2}(\partial_\mu\varphi)^2 - 2\zeta\cos\left(2\pi g_m\varphi\right)\right] . \tag{2.3}$$

[1] From now on, all the investigations will be performed in the Euclidean space-time.

Here, $\zeta \propto \exp\left(-\text{const}\, g_m^2\right)$ is the fugacity of a single magnetic object, $[\zeta] = (\text{mass})^4$, and the Debye mass of the dual Kalb-Ramond field φ, stemming from the expansion of the cosine, reads $m = 2\pi g_m \sqrt{2\zeta}$.

One can further define in the theory (2.1) an analogue of the Wilson loop at the free Kalb-Ramond fields:

$$W_{\text{free}}(\Sigma) = \exp\left(ig \int\limits_{\Sigma} d\sigma_{\mu\nu} h_{\mu\nu}\right) = \exp\left(\frac{ig}{2} \int\limits_{S} dS_{\mu\nu\lambda} H_{\mu\nu\lambda}\right). \tag{2.4}$$

Here, Σ is the world sheet of a certain closed electric string, and the integration in the last integral on the r.h.s. of eq. (2.4) is performed over some hypersurface S enclosed by Σ. Also, in eq. (2.4), g denotes the electric coupling constant, so that with 2π extracted explicitly on the r.h.s. of eq. (2.2), it obeys the following analogue of the Dirac quantization condition: $gg_m = n/3$. Here n is a certain integer, and in what follows we shall for simplicity restrict ourselves to the case $n = 1$. Next, setting in eq. (2.1) for a while $H^m_{\mu\nu\lambda} = 0$, we get

$$\langle W_{\text{free}}(\Sigma)\rangle = \exp\left[-g^2 \int\limits_{\Sigma} d\sigma_{\mu\nu}(x) \int\limits_{\Sigma} d\sigma_{\mu\nu}(y) D_0(x-y)\right], \tag{2.5}$$

where $D_0(x) \equiv \left(4\pi^2 x^2\right)^{-1}$ is the massless propagator. The contribution of magnetic objects to the analogue of the Wilson loop is defined by virtue of the last equality on the r.h.s. of eq. (2.4) as follows:

$$\langle W_{\text{m}}(\Sigma)\rangle = \left\langle \exp\left(\frac{ig}{2} \int\limits_{S} dS_{\mu\nu\lambda} H^m_{\mu\nu\lambda}\right)\right\rangle_{\text{gas}}, \tag{2.6}$$

Owing to eq. (2.2), it can be rewritten as $\prod\limits_{a=1}^{N} \exp\left[\frac{3igg_m}{\pi} q_a \eta(z_a)\right]$, where

$$\eta(x) = 2\pi^2 \partial^x_\mu \int\limits_{S} dS_\mu(y) D_0(x-y) \tag{2.7}$$

is the solid angle under which the hypersurface S is seen by an observer located at the point x, and $dS_\mu \equiv (1/6)\varepsilon_{\mu\nu\lambda\rho} dS_{\nu\lambda\rho}$. As it has already been mentioned, this hypersurface is a world volume of the confining membrane. In particular for a closed hypersurface, $\eta(x) = 2\pi^2$, and because of the Dirac quantization condition, $\langle W_{\text{m}}(\Sigma)\rangle$ becomes equal to unity. This fact proves that $\langle W_{\text{m}}(\Sigma)\rangle$ is independent of S, provided that the Dirac quantization condition holds. Indeed, suppose that this is not the case and consider the ratio of two $\langle W_{\text{m}}(\Sigma)\rangle$'s, defined at different hypersurfaces, S_1 and S_2. Then, the result has the form of the r.h.s. of eq. (2.6) with the integration performed

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over the closed hypersurface with the boundary $S_1 \cup S_2$. Therefore, such a ratio is equal to unity, which proves the desired hypersurface independence of $\langle W_{\rm m}(\Sigma)\rangle$. Clearly, such a consideration is analogous to the proof [6] of the fact that in compact QED with monopoles quantized according to the Dirac quantization condition, the monopole part of the Wilson loop does not depend on the shape of a surface bounded by the contour of the loop.

To construct the theory of confining membranes, it is useful to represent the statistical weight corresponding to the kinetic term in the action (2.3) as follows:[2]

$$\exp\left[-\frac{1}{2}\int d^4x(\partial_\mu\varphi)^2\right] =$$
$$= \int \mathcal{D}\rho \exp\left[-2\pi^2 g_m^2 \int d^4x d^4y \rho(x) D_0(x-y)\rho(y) - 2\pi g_m i \int d^4x \varphi\rho\right].$$

One can see that upon such a substitution, the field φ starts playing the rôle of the Lagrange multiplier, which is used for the exponentiation of the δ-function in the expression $1 = \int \mathcal{D}\rho\delta(\rho - \rho_{\rm gas})$. Such a unity, being inserted into the summation over the grand canonical partition function, represents the Legendre transformation from the dual Kalb-Ramond field φ to the dynamical density of magnetic objects, ρ. Once the kinetic term of the field φ is removed by the above substitution, this field can further be integrated out in the saddle-point approximation. This yields the representation for the partition function $\mathcal{Z} = \int \mathcal{D}\varphi \exp(-S_{\rm SG})$ in terms of dynamical densities: $\mathcal{Z} = \int \mathcal{D}\rho \exp(-S[\rho])$. The action $S[\rho]$ contains the Coulomb interaction of magnetic objects and their potential:

$$S[\rho] = 2\pi^2 g_m^2 \int d^4x d^4y \rho(x) D_0(x-y)\rho(y) + V[\rho]\,. \tag{2.8}$$

Here, the potential reads

$$V[\rho] = \int d^4x \left[\rho\, \mathrm{arcsinh}\varrho - 2\zeta\sqrt{1+\varrho^2}\right] \tag{2.9}$$

and $\varrho \equiv \rho/(2\zeta)$. In the dilute-gas approximation, $|\rho| \ll \zeta$, $V[\rho] \to \zeta \int d^4x \varrho^2$, and the integration over densities becomes gaussian. This leads to the following expression for the generating functional of ρ's in this limit:

$$\mathcal{Z}[j] = \exp\left[-\zeta \int d^4x d^4y j(x) j(y) \partial^2 D_m(x-y)\right]. \tag{2.10}$$

Here, $D_m(x) \equiv mK_1(m|x|)/(4\pi^2|x|)$ is the massive propagator with K_1 standing for the modified Bessel function. To get the expression for the contribution of magnetic objects to the analogue of the Wilson loop,

$$\langle W_{\rm m}(\Sigma)\rangle = \mathcal{Z}\left[\frac{i\eta}{\pi}\right], \tag{2.11}$$

[2]We omit inessential normalization constants implying that those are included in the respective integration measures.

it is useful to employ the formula

$$\partial_\mu \eta(x) = \pi^2 \varepsilon_{\mu\nu\lambda\rho} \left[\int\limits_\Sigma d\sigma_{\nu\lambda}(y) \partial_\rho^x D_0(x-y) - \frac{1}{3} \int\limits_S dS_{\nu\lambda\rho}(y) \delta(x-y) \right].$$

Introducing the analogue of the full Wilson loop, $\langle W(\Sigma)\rangle = \langle W_{\rm free}(\Sigma)\rangle \langle W_{\rm m}(\Sigma)\rangle$, we obtain the following membrane action:

$$-\ln\langle W(\Sigma)\rangle = 4\pi^2 \zeta \int\limits_S dS_\mu(x) \int\limits_S dS_\mu(y) D_m(x-y) + \\ + \frac{g^2}{4} \int\limits_\Sigma d\sigma_{\mu\nu}(x) \int\limits_\Sigma d\sigma_{\mu\nu}(y) \left[9D_m(x-y) - 5D_0(x-y)\right]. \quad (2.12)$$

As it was proved after eq. (2.7), $\langle W_{\rm m}(\Sigma)\rangle$ is independent of S. Analogously to the theory of confining strings, the S-dependence of the r.h.s. of eq. (2.12), which appears in the dilute-gas approximation, becomes eliminated upon the summation over branches of the arcsinh-function standing in the full potential (2.9). This is the essence of correspondence between fields and membranes in the model under study.

Note that all the above considerations can be performed in terms of the field $\mathcal{H}_{\mu\nu\lambda}$, $\varepsilon_{\mu\nu\lambda\rho}\partial_\mu \mathcal{H}_{\nu\lambda\rho} \propto \rho$. Being rewritten through this field, the membrane representation of $\langle W(\Sigma)\rangle$, given by eq. (2.11) with the generating functional defined w.r.t. the full action (2.8)–(2.9), is a direct generalization of the theory of confining strings. Indeed, the latter one describes the string representation of the Wilson loop in 3D compact QED in the language of the Kalb-Ramond field $h_{\mu\nu}$, $\varepsilon_{\mu\nu\lambda}\partial_\mu h_{\nu\lambda} \propto \rho$, where ρ is the dynamical monopole density.

Next, we see that the first term on the r.h.s. of eq. (2.12) is responsible for confinement of closed strings. One can parametrize the world volume of the membrane, S, by the vector $x_\mu(\xi)$, $\xi = (\xi^1, \xi^2, \xi^3)$, and expand the above-mentioned term in powers of the derivatives $\partial_a \equiv \partial/\partial\xi^a$ supplied with the appropriate powers of m in the denominator. Such an expansion is similar to the analogous expansion of non-local string effective actions [7, 8, 4], and its first three leading terms have the following form:

$$\zeta \int\limits_S dS_\mu(x) \int\limits_S dS_\mu(y) D_m(x-y) \simeq \sigma \int d^3\xi \sqrt{\hat g} + \alpha \int d^3\xi \sqrt{\hat g} \left(K^{ia}{}_a\right)^2 + \beta \int d^3\xi \sqrt{\hat g} \mathcal{R}. \quad (2.13)$$

Here, $\hat g^{ab}(\xi) = (\partial^a x_\mu(\xi))(\partial^b x_\mu(\xi))$ is the induced metric tensor on S, $\hat g \equiv \det ||\hat g^{ab}||$, and $K^{ia}{}_b$ is the second fundamental form of S defined according to the Gauss-Weingarten formula:

$$D_a D_b x_\mu(\xi) = \left(\partial_a \partial_b - \Gamma^c_{ab}\partial_c\right) x_\mu(\xi) = K^i_{ab} n^i_\mu.$$

Here, Γ^c_{ab} denotes the Christoffel symbol corresponding to the metric $\hat g^{ab}(\xi)$, and

n^i_μ's, $i = 1, 2, 3$, are the unit normal vectors to S: $n^i_\mu \partial_a x_\mu(\xi) = 0$, $n^i_\mu n^j_\mu = \delta^{ij}$. Also in eq. (2.13), $\mathcal{R}$ stands for the scalar curvature corresponding to the metric $\hat{g}^{ab}(\xi)$, which is related to the second fundamental form as $\mathcal{R} = \left(K^{ia}{}_a\right)^2 - K^{ia}{}_b K^{ib}{}_a$. As far as the coupling constants on the r.h.s. of eq. (2.13) are concerned, those up to certain dimensionless factors have the form:

$$\sigma \propto \frac{\zeta}{m^3} \int d^3 z D_m\left(z^2\right) \propto \frac{\zeta}{m}, \quad \alpha \propto \beta \propto \frac{\zeta}{m^5} \int d^3 z z^2 D_m\left(z^2\right) \propto \frac{\zeta}{m^3},$$

where $D_m\left(z^2\right) \equiv m^2 K_1(|z|)/(4\pi^2 |z|)$. The first term on the r.h.s. of eq. (2.13) is the standard membrane action [9]. It is analogous to the Nambu-Goto term in the action of confining strings and expresses confinement of a closed electric string in the sense of the volume law. As it has already been mentioned in the Introduction, the latter is a generalization of the Wilson area law for point-like particles to the case of strings. The second term in the expansion (2.13) describes the stiffness of a membrane [5, 10]. Note also that contrary to the case of confining strings, the last term on the r.h.s. of eq. (2.13) is not a full derivative, since the membrane action is three-dimensional.

The second term on the r.h.s. of eq. (2.12) is analogous to the perimeter term for the Wilson loop in 3D compact QED. However, the difference between the two theories is that in the model (2.1), contrary to compact QED, the contribution of the free Kalb-Ramond fields (2.5) does not exactly cancel out with the respective massless contribution to $\langle W(\Sigma)\rangle$ stemming from the magnetic objects. However, we see that for small enough Σ's, namely for those whose typical sizes do not exceed $1/m$, $\langle W(\Sigma)\rangle \to \langle W_{\rm free}(\Sigma)\rangle$, as it should be. In the opposite case of Σ's whose sizes are much larger than $1/m$, the first term on the r.h.s. of eq. (2.12) dominates over the second one, and the regime of the system under study becomes purely confining.

The above-presented considerations can straightforwardly be translated to the SU(3)-inspired case along with the lines of ref. [11]. At this point, the following remark is in order. The SU(3)-inspired generalization of 3D compact QED [12] can be viewed as a $[\mathrm{U}(1)]^2$ gauge theory with monopoles, stemming from the 3D SU(3) Georgi-Glashow model. However, even in 3D, there does not exist a clear way to extend the usual SU(2) Georgi-Glashow model to a certain SU(2)-theory, which might lead to the continuous action (2.1). Therefore, the action (2.1) should be understood only as a theory yielding upon the dualization a continuous description of the lattice Coulomb gas (1.1). In the same sense, the SU(3)-inspired theory, we are going to discuss below, describes a continuous formulation of the SU(3)-version of the lattice model (1.1).

In the model under consideration, the charges of magnetic objects are distributed over the respective root lattice defined by the vectors

$$\vec{q}_1 = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right), \quad \vec{q}_2 = (-1, 0), \quad \vec{q}_3 = \left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right), \quad \vec{q}_{-i} = -\vec{q}_i.$$

The density $\rho_{\rm gas}$ becomes modified to $\vec\rho_{\rm gas}(x) = \sum\limits_{a=1}^{N} \vec q_{i_a}\delta(x-z_a)$, and accordingly the fields $h_{\mu\nu}$ (as well as its field strength tensor), $H^m_{\mu\nu\lambda}$, and φ become two-component vectors. The action (2.3) takes the form

$$S_{\rm SG}^{\rm SU(3)} = \int d^4x \left[\frac{1}{2}\left(\partial_\mu\vec\varphi\right)^2 - 2\zeta\sum_{i=1}^{3}\cos\left(2\pi g_m\vec q_i\vec\varphi\right)\right], \tag{2.14}$$

which yields the Debye mass $m_{\rm SU(3)} = 2\pi g_m\sqrt{3\zeta}$. With the replacement $\rho\to\vec\rho$, the representation of the action in terms of densities of magnetic objects, eq. (2.8), keeps its form, where now

$$V[\vec\rho] \equiv \sum_{i=1}^{3} V\left[\rho_i\right].$$

In the latter formula, $V[\rho_i]$ is defined by eq. (2.9) with ρ_i's related to the components ρ^1 and ρ^2 of the vector $\vec\rho$ as follows (see ref. [11] for details): $\rho_1 \equiv \left(\rho^1/\sqrt{3}+\rho^2\right)/\sqrt{3}$, $\rho_2 \equiv -2\rho^1/3$, $\rho_3 \equiv \left(\rho^1/\sqrt{3}-\rho^2\right)/\sqrt{3}$. In the dilute-gas approximation, $|\vec\rho| \ll \zeta$, the potential $V[\vec\rho]$ becomes quadratic, $V[\vec\rho] \to [1/(6\zeta)]\int d^4x\vec\rho^{\,2}$. This yields for the generating functional $\mathcal{Z}$ of correlators of $\vec\rho$'s an expression of the form (2.10) with the replacements $j\to\vec j$, $\zeta\to 3\zeta/2$, and $m\to m_{\rm SU(3)}$.

The free part of the analogue of the Wilson loop is defined as follows [cf. eq. (2.4)]:

$$\begin{aligned} W_{\rm free}(\Sigma) &\equiv \frac{1}{3}\,{\rm tr}\,\exp\left(ig\int\limits_\Sigma d\sigma_{\mu\nu}\vec h_{\mu\nu}\frac{\vec\lambda}{2}\right) = \frac{1}{3}\sum_{\alpha=1}^{3}\exp\left(ig\vec\mu_\alpha\int\limits_\Sigma d\sigma_{\mu\nu}\vec h_{\mu\nu}\right) = \\ &= \frac{1}{3}\sum_{\alpha=1}^{3}\exp\left(\frac{ig\vec\mu_\alpha}{2}\int\limits_S dS_{\mu\nu\lambda}\vec H_{\mu\nu\lambda}\right). \end{aligned} \tag{2.15}$$

In this equation, $\vec\lambda = (\lambda^3,\lambda^8)$ are the diagonal Gell-Mann matrices, and $\vec\mu_\alpha$'s stand for the weight vectors of the group SU(3), $\vec\mu_1 = \left(-\frac{1}{2},\frac{1}{2\sqrt{3}}\right)$, $\vec\mu_2 = \left(\frac{1}{2},\frac{1}{2\sqrt{3}}\right)$, $\vec\mu_3 = \left(0,-\frac{1}{\sqrt{3}}\right)$. Averaging eq. (2.15) over the free Kalb-Ramond field $\vec h_{\mu\nu}$ we arrive at eq. (2.5) with the replacement $g^2\to g^2/6$.

Similarly to eq. (2.6), by virtue of the last equality in eq. (2.15) one can define the contribution of magnetic objects to the analogue of the Wilson loop:

$$\left\langle W_m(\Sigma)\right\rangle \equiv \frac{1}{3}\sum_{\alpha=1}^{3}\left\langle\exp\left(\frac{ig\vec\mu_\alpha}{2}\int\limits_S dS_{\mu\nu\lambda}\vec H^m_{\mu\nu\lambda}\right)\right\rangle_{\rm gas} = \frac{1}{3}\sum_{\alpha=1}^{3}\prod_{a=1}^{N}\exp\left[\frac{2i}{\pi}\vec\mu_\alpha\vec q_{i_a}\eta(z_a)\right].$$

In the last equality, we have imposed the modified, SU(3)-one, Dirac quantization condition $gg_m = 2n/3$, where n is again some integer which we have set for simplicity equal to unity. Next, since for any α and i_a, the scalar product $\vec\mu_\alpha\vec q_{i_a}$ is equal either to

0 or to $\pm 1/2$, $\langle W_m(\Sigma)\rangle$ for a closed hypersurface S is equal to unity. This again proves that $\langle W_m(\Sigma)\rangle$ is independent of the choice of S [cf. the discussion after eq. (2.7)]. The SU(3)-version of the theory of confining membranes is then given by the following expression for $\langle W_m(\Sigma)\rangle$:

$$\langle W_m(\Sigma)\rangle = \frac{1}{3}\sum_{\alpha=1}^{3} \mathcal{Z}\left[\frac{2i\vec{\mu}_\alpha}{\pi}\eta\right].$$

Here, the average in $\mathcal{Z}$'s is performed over the full action written in terms of $\vec{\rho}$'s, which was defined after eq. (2.14). Similarly to the SU(2)-inspired case, we obtain in the dilute-gas approximation the nonlocal membrane action from the analogue of the full Wilson loop, $\langle W(\Sigma)\rangle = \langle W_{\rm free}(\Sigma)\rangle\langle W_{\rm m}(\Sigma)\rangle$. It reads

$$-\ln\langle W(\Sigma)\rangle = 8\pi^2\zeta \int\limits_S dS_\mu(x)\int\limits_S dS_\mu(y) D_{m_{\rm SU(3)}}(x-y) + \\ +\frac{g^2}{4}\int\limits_\Sigma d\sigma_{\mu\nu}(x)\int\limits_\Sigma d\sigma_{\mu\nu}(y)\left[3D_{m_{\rm SU(3)}}(x-y) - \frac{7}{3}D_0(x-y)\right]. \quad (2.16)$$

Apparently, one can perform the same derivative expansion of the first term on the r.h.s. of eq. (2.16) as the one performed in the SU(2)-inspired case, and the result for the leading terms of such an expansion has the form of the r.h.s. of eq. (2.13). Notice also that as it should be, for small Σ's, whose typical sizes do not exceed $1/m_{\rm SU(3)}$, $\langle W(\Sigma)\rangle \to \langle W_{\rm free}(\Sigma)\rangle$.

3. Dimensional reduction in the theory which confines closed strings and point-like charges

As a concrete example of the theory with closed electric strings let us consider dual abelian Higgs model,[3] where those are the dual Abrikosov-Nielsen-Olesen strings [13]. This is also an example of the model which confines external point-like electric charges if there inserted any, just upon the formation of dual strings between those. Let us consider this model in the London limit, i.e. the limit of infinitely heavy dual Higgs field, which means that the strings have infinitely thin cores. In this limit, the action of the model reads

$$S_{\rm DAHM} = \int d^4x \left[\frac{1}{4}F_{\mu\nu}^2 + \frac{\eta^2}{2}(\partial_\mu\theta - 2\bar{g}_m B_\mu)^2\right]. \quad (3.1)$$

Here, $F_{\mu\nu}$ is the field strength tensor of the dual vector potential B_μ, η is the v.e.v. of the dual Higgs field, and $\bar{g}_m$ is the dimensionless magnetic coupling constant.[4] The

[3]Below we consider for simplicity the SU(2)-inspired theory of confining membranes.

[4]The doubled magnetic charge of the dual Higgs field in eq. (3.1) is inspired by the analogy with the 3D (dual) Landau-Ginzburg superconductor [14, 15]. In the latter case, this is nothing but the charge of the (magnetic) Cooper pair.

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phase of the dual Higgs field, θ, can be decomposed into the sum of a single- and multivalued parts. The latter one is unambiguously related to the vorticity tensor current $\bar{\Sigma}_{\mu\nu}(x) \equiv \int\limits_{\Sigma} d\sigma_{\mu\nu}(x(\sigma))\delta(x - x(\sigma))$ of the closed dual strings. In this formula, the vector $x_\mu(\sigma)$, $\sigma \equiv (\sigma^1, \sigma^2)$, parametrizes the string world sheet Σ. The relation between the multivalued part of θ and $\bar{\Sigma}_{\mu\nu}$ can be expressed by means of the following equation [15, 16] obeyed by θ:

$$(\partial_\mu \partial_\nu - \partial_\nu \partial_\mu)\theta = \pi \varepsilon_{\mu\nu\lambda\rho} \bar{\Sigma}_{\lambda\rho} \,. \tag{3.2}$$

This correspondence enables one to derive the string representation [17, 18] of the dual abelian Higgs model. Namely, one can transform the action (3.1) into the form

$$S_{\rm DAHM} = (\pi\eta)^2 \int d^4x d^4y \bar{\Sigma}_{\mu\nu}(x) D_{m_B}(x - y) \bar{\Sigma}_{\mu\nu}(y) \,, \tag{3.3}$$

where $m_B = 2\bar{g}_m \eta$ is the mass of the dual vector field, acquired by it due to the Higgs mechanism.

In the representation (3.3), dual strings are considered as individual ones, and to construct the partition function one should supply this action with a certain prescription of the summation over string world sheets. One concrete example of such a prescription stems from the known fact [19] that at low temperatures, closed dual strings with minimal opposite winding numbers couple to each other and form virtual bound states, the so-called vortex loops. Since these objects are virtual and therefore small-sized, they can naturally be treated in the dilute-gas approximation. Being rewritten in terms of dynamical densities $\Sigma_{\mu\nu}$'s of these loops, the partition function of the dual abelian Higgs model takes the following form [20]:

$$\mathcal{Z}_{\rm DAHM} = \int \mathcal{D}\Sigma_{\mu\nu} \exp\left\{ - \left[(\pi\eta)^2 \int d^4x d^4y \Sigma_{\mu\nu}(x) D_{m_B}(x - y) \Sigma_{\mu\nu}(y) + \right.\right.$$
$$\left.\left. + V\left[\Lambda^2 \sqrt{\Sigma_{\mu\nu}^2}\right]\right]\right\}. \tag{3.4}$$

In this formula, $\Lambda = \sqrt{L/a^3}$ is an UV momentum cutoff, where a and L denote respectively typical sizes of vortex loops and characteristic distances between them in the gas. In the dilute-gas approximation under study, $a \ll L$ and $\Lambda \gg a^{-1}$. Next, in eq. (3.4), the potential of vortex loops, V, is given by eq. (2.9) with the replacement $\zeta \to \bar{\zeta}$. Here, $\bar{\zeta} \propto {\rm e}^{-S_0}$ denotes the fugacity of a single vortex loop, $\left[\bar{\zeta}\right] = ({\rm mass})^4$, where S_0 is the action of a single loop estimated in ref. [20]. In the dilute-gas approximation under study, $\sqrt{\Sigma_{\mu\nu}^2} \ll \bar{\zeta}/\Lambda^2$, the potential again becomes a quadratic functional: $V \to \left[\Lambda^4/(4\bar{\zeta})\right] \int d^4x \Sigma_{\mu\nu}^2$.

The bilocal correlator of densities of the vortex loops can then be calculated by the evaluation of the respective gaussian integral [cf. the generating functional (2.10)] and in addition by taking explicitly into account the closeness of these objects with the use of the Hodge decomposition theorem (see the last ref. of [20] for details). In what follows, we shall be interested in the limit when the v.e.v. of the dual Higgs field, η, is very large. In that limit, the action of the grand canonical ensemble of vortex loops standing in the exponent on the r.h.s. of eq. (3.4) becomes local and takes the form: $c\int d^4x\Sigma_{\mu\nu}^2$, where $c\equiv\left(\pi^2/\bar{g}_m^2+\Lambda^4/\bar{\zeta}\right)/4$.[5] The above-mentioned bilocal correlator of densities of the vortex loops corresponding to this action reads:

$$\left<\Sigma_{\mu\nu}(x)\Sigma_{\lambda\rho}(0)\right>=-\frac{1}{4c}\varepsilon_{\mu\nu\alpha\beta}\varepsilon_{\lambda\rho\gamma\beta}\partial_\alpha\partial_\gamma D_0(x)\,. \tag{3.5}$$

Clearly, it respects the closeness of vortex loops, $\partial_\mu\Sigma_{\mu\nu}=0$. The higher $2n$-point correlators of $\Sigma_{\mu\nu}$'s can be obtained from the bilocal one by means of the Wick's theorem.

For our further purposes, we find it convenient to pass from the correlator (3.5) to the bilocal correlator of "hypersurfaces" "enclosed" by the density of vortex loops, $\Sigma_{\mu\nu}(x)$. The quotation marks here clearly mean that these "hypersurfaces" are some fields, S_μ's, corresponding to $\Sigma_{\mu\nu}$'s according to the law [cf. the remark after eq. (3.9) below]

$$\varepsilon_{\mu\nu\lambda\rho}\Sigma_{\lambda\rho}\to\partial_\mu S_\nu\,. \tag{3.6}$$

The bilocal correlator of such fields is then very simple and reads

$$\left<S_\mu(x)S_\nu(0)\right>=\frac{1}{4c}\delta_{\mu\nu}D_0(x)\,. \tag{3.7}$$

Let us now consider the model, described by the sum of the actions (2.3) and (3.1) at $\eta\to\infty$, which confines both point-like electric charges and closed dual strings. The interaction of vortex loops with $H_{\mu\nu\lambda}^m$, described by eq. (2.6), can be rewritten in terms of the field φ by virtue of the correspondence

$$H_{\mu\nu\lambda}^m\to -i\varepsilon_{\mu\nu\lambda\rho}\partial_\rho\varphi\,. \tag{3.8}$$

This yields for the partition function of such a model the following expression:

$$\mathcal{Z}=\left<\int\mathcal{D}\varphi\exp\left[-S_{\rm SG}-\frac{g}{2\pi^2}\int d^4x\varphi\partial^2\eta\right]\right>.$$

[5]Note that the term $(\pi/2\bar{g}_m)^2\int d^4x\Sigma_{\mu\nu}^2$ in this action, which does not depend on the particular properties of the ensemble of vortex loops, can be obtained directly from the initial action (3.1) by taking in it the limit $\eta\to\infty$. Indeed, one has:

$$\exp\left[-\frac{\eta^2}{2}\int d^4x\left(\partial_\mu\theta-2\bar{g}_mB_\mu\right)^2\right]=\int\mathcal{D}\lambda_\mu\exp\left\{-\int d^4x\left[\frac{1}{2\eta^2}\lambda_\mu^2+i\lambda_\mu\left(\partial_\mu\theta-2\bar{g}_mB_\mu\right)\right]\right\}\stackrel{\eta\to\infty}{\longrightarrow}$$
$$\stackrel{\eta\to\infty}{\longrightarrow}\delta\left(\partial_\mu\theta-2\bar{g}_mB_\mu\right).$$

Integrating further in the partition function over the B_μ-field, one gets for the action (3.1) at $\eta\to\infty$: $(4\bar{g}_m)^{-2}\int d^4x\left[(\partial_\mu\partial_\nu-\partial_\nu\partial_\mu)\theta\right]^2$. By virtue of eq. (3.2) this yields the announced result.

Here, the average is performed over the grand canonical ensemble of vortex loops, and $\eta(x)$ is the "solid angle" of the vortex loop, i.e. the field related to S_μ by the equation

$$\partial_\mu \eta = -2\pi^2 S_\mu \,. \tag{3.9}$$

Note that the proof of eq. (3.9) becomes straightforward by acting on both sides of this equation with the operator ∂_μ and further by performing for a while the inverse Legendre transformation from the dynamical "hypersurfaces" S_μ's to the real ones, corresponding to the gas of vortex loops. The latter read $S_\mu^{\rm gas}(x) = \sum\limits_{a=1}^{N} n_a \int dS_\mu(x_a(\xi))\delta(x-x_a(\xi))$, where $n_a = \pm 1$ denote the winding numbers of vortex loops. The desired proof then stems from the definition of the solid angle (2.7). Clearly, the correspondences (3.6) and (3.8) can be proved by means of the same inverse Legendre transformation.

Introducing further the notation $g(2\pi^2)^{-1}\partial^2\eta \equiv h$ we get from eqs. (3.7) and (3.9) the bilocal correlator of h's: $\langle h(x)h(0)\rangle = A\delta(x)$, where $A \equiv g^2/(4c)$. Finally, rescaling the fields as $\varphi^{\rm new} = \varphi/\sqrt{A}$, $h^{\rm new} = h/\sqrt{A}$, we arrive at the following expression for the partition function of our model (we skip further for brevity the superscription "new"):

$$\mathcal{Z} = \left\langle \int \mathcal{D}\varphi \exp\left\{-A\int d^4x \left[\frac{1}{2}(\partial_\mu\varphi)^2 - 2\zeta A^{-1}\cos\left(\frac{\pi\varphi}{\sqrt{c}}\right) + h\varphi\right]\right\}\right\rangle_{\frac{1}{2}\int d^4x h^2} . \tag{3.10}$$

One can now (as it further be argued, naively) see that owing to the coupling of the dual Kalb-Ramond field φ to the stochastic gaussian field h (analogous to the random external magnetic field in the case of Ising model), the model (3.10) undergoes the dimensional reduction to two dimensions. The proof of this statement is similar to that of ref. [21]. The only difference of the case under study from the one considered in ref. [21] is that our potential of the field φ has the form

$$\mathcal{V} = -2\zeta A^{-1}\cos\left(\frac{\pi\varphi}{\sqrt{c}}\right), \tag{3.11}$$

rather than that of the massive φ^4-theory. However, this difference is not crucial. Indeed, one can consider for instance the bilocal correlator of the mean field φ_h, a solution to the classical equation of motion in the theory (3.10),

$$\partial^2\varphi_h - \mathcal{V}'(\varphi_h) = h\,, \tag{3.12}$$

which minimizes the action. It reads (see ref. [21] for details):

$$\langle\varphi_h(x_1)\varphi_h(x_2)\rangle_{\frac{1}{2}\int d^4x h^2} = \int \mathcal{D}\varphi\mathcal{D}\alpha\mathcal{D}\bar\psi\mathcal{D}\psi \exp\left(-\int d^4x\mathcal{L}\right)\varphi(x_1)\varphi(x_2)\,. \tag{3.13}$$

Here,

$$\mathcal{L} = \bar{\psi}\left[-\partial^2 + \mathcal{V}''(\varphi)\right]\psi + \alpha\left[-\partial^2\varphi + \mathcal{V}'(\varphi)\right] - \frac{1}{2}\alpha^2 \tag{3.14}$$

and ψ, $\bar{\psi}$ are the fermionic fields which represent the jacobian det $|\delta h/\delta\varphi|$. Note that the proof of eqs. (3.13) and (3.14) is straightforward upon the introduction into the average on the l.h.s. of eq. (3.13) of the following unity:

$$\begin{aligned} 1 &= \int \mathcal{D}h\delta\left[h - \left(\partial^2\varphi - \mathcal{V}'(\varphi)\right)\right] = \\ &= \int \mathcal{D}\varphi\mathcal{D}\bar{\psi}\mathcal{D}\psi \exp\left[-\int d^4x\bar{\psi}\left(-\partial^2 + \mathcal{V}''(\varphi)\right)\psi\right]\delta\left[h - \left(\partial^2\varphi - \mathcal{V}'(\varphi)\right)\right]. \end{aligned}$$

Next, similarly to the case of the potential of the massive φ^4-theory, it is straightforward to prove that the lagrangian (3.14) with the potential $\mathcal{V}$ of the general form

$$\mathcal{V} = \sum_{n=1}^{\infty} c_n\varphi^{2n} \tag{3.15}$$

[and thus, in particular, with our potential (3.11)] is invariant under the following supersymmetry transformation:

$$\delta\varphi = -\varepsilon_\mu x_\mu\psi\,, \quad \delta\alpha = 2\varepsilon_\mu\partial_\mu\psi\,, \quad \delta\psi = 0\,, \quad \delta\bar{\psi} = \varepsilon_\mu x_\mu\alpha + 2\varepsilon_\mu\partial_\mu\varphi\,, \tag{3.16}$$

where ε_μ stands for an infinitesimal anticommuting vector. Also, similarly to ref. [21], one can introduce a superfield

$$\Phi\left(x,\theta,\bar{\theta}\right) = \varphi + \bar{\theta}\psi + \bar{\psi}\theta + \theta\bar{\theta}\alpha\,,$$

where the integration over the anticommuting variables θ and $\bar{\theta}$ is defined as

$$\int d\bar{\theta}d\theta\theta\bar{\theta} = 1\,.$$

Furthermore, one can introduce the superspace laplacian $D^2 = \partial^2 + \partial^2/(\partial\bar{\theta}\partial\theta)$, where derivatives act on the left. With these definitions, it is possible to prove the following equality

$$\int d^4x\mathcal{L} = \int d^4xd\bar{\theta}d\theta\left[-\frac{1}{2}\Phi D^2\Phi + \mathcal{V}(\Phi)\right], \tag{3.17}$$

valid for an arbitrary potential of the form (3.15). Such a proof becomes straightforward with the use of the known formula

$$\left(\sum_{i=1}^{5} a_i\right)^n = \sum_{\substack{k_1+\ldots+k_5=n \\ k_i\geq 0}} \frac{n!}{k_1!\ldots k_5!}a_1^{k_1}\ldots a_5^{k_5}\,,$$

where a_i's represent the terms of Φ^2,

$$\Phi^2 = \varphi^2 + 2\bar{\theta}\psi\varphi + 2\bar{\psi}\theta\varphi + 2\theta\bar{\theta}\varphi\alpha + 2\theta\bar{\theta}\bar{\psi}\psi\,.$$

After that, one should only pick up the terms surviving the integration over $\bar{\theta}$, θ.

Once the supersymmetric formulation of the action, eq. (3.17), is found, further proof of the dimensional reduction is identical to that of ref. [21]. Indeed, the action (3.17) is manifestly invariant under rotations in superspace, which is alternatively expressed by the invariance of the lagrangian (3.14)–(3.15) under the supersymmetry transformations (3.16). Such rotations leave the interval in superspace, $\left(x^2 + \theta\bar{\theta}\right)$, invariant. The desired proof then completes itself by noting that for any supersymmetrically invariant function, $f = f\left(x^2 + \theta\bar{\theta}\right)$, the following equality holds:

$$\int d^4x d\bar{\theta} d\theta f\left(x^2 + \theta\bar{\theta}\right) = -\pi \int d^2x f\left(x^2\right).$$

The considerations presented above can straightforwardly be translated to an arbitrary $2n$-point correlator of φ_h's. Thus, we conclude that all $2n$-point correlators of the field φ_h in the theory (3.10) are equal to the respective correlators of the field φ computed in the same theory without the stochastic field h and in two dimensions lower. This means that our 4D model confining point-like charges and closed dual strings should be described by the 2D sine-Gordon theory of the scalar field. The latter one is known as an example of the completely integrable field theory (see e.g. ref. [22] and references therein). In particular, at a certain critical temperature, the 2D Coulomb gas described by this theory undergoes the Berezinskii-Kosterlitz-Thouless phase transition from the molecular to the plasma phase [23]. Such a phase transition occurs when the mean squared separation between two objects forming a molecule becomes infinite. This corresponds to the divergency of the integral

$$\left\langle r^2\right\rangle \sim \int d^2x|x|^2 \exp\left[-\frac{\ln(\mu|x|)}{2\pi T}\frac{\pi^2}{c}\right] \sim \int^{\infty} d|x||x|^{3-\frac{\pi}{2cT}}\,,$$

where μ denotes the IR cutoff. The respective critical temperature (which is clearly dimensionless, as it should be in 2D [23]), above which the integral diverges at the upper limit, reads

$$T_{\rm cr} = \frac{\pi}{8c} = \frac{\pi}{2}\left(\frac{\pi^2}{\bar{g}_m} + \frac{\Lambda^4}{\bar{\zeta}}\right)^{-1}. \tag{3.18}$$

Note that this temperature is clearly much smaller than unity, since the UV cutoff Λ is large. (Another reason for the smallness of $T_{\rm cr}$ can occur if we consider the case when $\bar{g}_m \ll 1$. Such a situation apparently parallels the London limit of the dual abelian Higgs model considered.)

However, there unfortunately exist indications that the above-discussed dimensional reduction to the full 2D sine-Gordon theory is rather naive. The point is that the above-presented considerations are strictly speaking valid only provided that there exists the unique solution to the saddle-point equation (3.12) minimizing the respective action [24]. Otherwise, a constraint, which is necessary to be inserted into the functional integral in order to eliminate other solutions, leads to the violation of eq. (3.13). For our action with the potential (3.11), the uniqueness of the saddle point can take place only under some approximation. Indeed, once such a solution is found, there apparently exists an infinite set of solutions following from that one upon the translations of it by $2\sqrt{c}n$, where $n = \pm 1, \pm 2, \ldots$. Moreover, within half a period from a given solution there always exists another one. The above-mentioned approximation then would mean the restriction of the range where φ is defined, from the whole real axis to the one half of the period, $(-\sqrt{c}/2, \sqrt{c}/2)$. Such a restriction which obviously eliminates the above-discussed extra saddle points is a good approximation, since $c \gg 1$ [cf. the discussion after eq. (3.18)]. However, even this approximation does not guarantee us the uniqueness of the desired saddle-point solution, since even in such a range, the 4D sine-Gordon equation may have many solutions.

The only rigorous statement concerning dimensional reduction in our case can be done if we approximate the sine-Gordon theory by the free bosonic one, i.e. set

$$\mathcal{V} \simeq \frac{\pi^2\zeta}{Ac}\varphi^2 = \frac{4\pi^2\zeta}{g^2}\varphi^2 .$$

This approximation is valid when g is large enough, i.e. in the strong-coupling limit of the theory (2.1). Clearly, in such a case the saddle-point equation (3.12) has the unique solution. Therefore, no additional constraints should be inserted into the integral over the φ-field, and the whole above discussion concerning dimensional reduction is valid. We thus conclude that the theory (2.1) in the strong-coupling limit, being coupled to the ensemble of vortex loops, undergoes the dimensional reduction to the 2D free bosonic theory.

4. Conclusions

In the present paper, we have considered the 4D model, which confines closed electric strings. This model is a natural generalization of 3D compact QED to a certain 4D theory. Such a theory describes the free Kalb-Ramond field and a Coulomb gas of point-like magnetically charged objects with a finite action. We have defined in that theory an analogue of the Wilson loop and constructed for it the respective generalization of confining strings to confining membranes. From the so-obtained theory of confining membranes we have in particular deduced confinement of closed electric strings, which is expressed in terms of the volume law for the analogue of the Wilson loop. We have also presented the generalization of this analysis to the SU(3)-inspired case.

Next, we have considered one concrete example of the theory possessing closed electric strings, namely the dual abelian Higgs model in the London limit. We have explored the combined theory, which confines both electric strings (due to the medium of point-like magnetic objects) and electric charges (due to the condensation of the dual Higgs field) in the limit when the v.e.v. of the dual Higgs field is infinitely large. In that case, the electric vortex loops of the dual abelian Higgs model play for the dual Kalb-Ramond field the same rôle as the random external gaussian magnetic field plays for the mean field in the Ising model. Owing to this fact, the system naively undergoes the dimensional reduction and becomes described by the 2D sine-Gordon theory. The latter one is known to be a completely integrable field theory, and many of its properties, such as the semiclassical spectrum, are known exactly. Besides that, this theory describes a 2D Coulomb gas, which at a certain critical temperature passes through the Berezinskii-Kosterlitz-Thouless phase transition from the molecular to the plasma phase. The critical temperature of this transition is expressed in terms of the parameters of the dual abelian Higgs model and turns out to be much smaller than unity. However, it is finally argued that the dimensional reduction is strictly speaking valid only in the case when the original theory of point-like magnetic objects is considered in the strong-coupling limit. In that limit, the dimensional reduction transforms the above-mentioned combined theory to a simple 2D free bosonic one.

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References

[1] A.M. Polyakov, *Gauge fields and strings*, Harwood Academic Publishers, Chur 1987.

[2] M. Kalb and P. Ramond, *Classical direct interstring action*, *Phys. Rev.* **D 9** (1974) 2273.

[3] M.C. Diamantini, *Duality and confinement in massive antisymmetric tensor gauge theories*, *Phys. Lett.* **B 512** (2001) 174 [hep-th/0104192].

[4] A.M. Polyakov, *Confining strings*, *Nucl. Phys.* **B 486** (1997) 23 [hep-th/9607049].

[5] H. Kleinert, *The membrane properties of condensing strings*, *Phys. Lett.* **B 174** (1986) 335.

[6] Y.A. Simonov, *Cluster expansion, nonabelian Stokes theorem and magnetic monopoles*, *Sov. J. Nucl. Phys.* **50** (1989) 134.

[7] H. Kleinert, *Dynamical generation of string tension and stiffness in strings and membranes*, *Phys. Lett.* **B 211** (1988) 151.

[8] D.V. Antonov, D. Ebert and Y.A. Simonov, *Curvature expansion for the background-induced gluodynamics string*, *Mod. Phys. Lett.* **A 11** (1996) 1905 [`hep-th/9605086`];
M.C. Diamantini, F. Quevedo and C.A. Trugenberger, *Confining strings with topological term*, *Phys. Lett.* **B 396** (1997) 115 [`hep-th/9612103`];
For a review see: D. Antonov, *String representation and nonperturbative properties of gauge theories*, *Surveys High Energ. Phys.* **14** (2000) 265 [`hep-th/9909209`].

[9] P.A. Collins and R.W. Tucker, *Classical and quantum mechanics of free relativistic membranes*, *Nucl. Phys.* **B 112** (1976) 150;
A. Sugamoto, *Theory of membranes*, *Nucl. Phys.* **B 215** (1983) 381.

[10] E. Elizalde and S.D. Odintsov, *Effective potential and stability of the rigid membrane*, *Phys. Rev.* **D 47** (1993) 2497 [`hep-th/9205050`];
E. Elizalde, S. Leseduarte and S.D. Odintsov, *Partition functions for the rigid string and membrane at any temperature*, *Phys. Rev.* **D 48** (1993) 1757 [`hep-th/9304071`].

[11] D. Antonov, *Confining strings in the* SU(3)*-gluodynamics*, *Europhys. Lett.* **52** (2000) 54 [`hep-th/0003043`]; *Confining strings in the abelian-projected* SU(3)*-gluodynamics II. 4D-case with θ-term*, *Europhys. Lett.* **54** (2001) 715 [`hep-ph/0012009`];
N. Agasian and D. Antonov, *Confining strings and RG flow in the (2+1)-dimensional Georgi-Glashow model and its* SU(3)*-generalization*, *J. High Energy Phys.* **06** (2001) 058 [`hep-th/0104029`].

[12] S.R. Wadia and S.R. Das, *Topology of quantum gauge fields and duality. 1. Yang-Mills Higgs system in (2+1) dimensions*, *Phys. Lett.* **B 106** (1981) 386; *Erratum-ibid.* **B 108** (1982) 435;
N.J. Snyderman, *The physics of dual vortices and static baryons in (2+1)- dimensions*, *Nucl. Phys.* **B 218** (1983) 381.

[13] A.A. Abrikosov, *On the magnetic properties of superconductors of the second group*, *Sov. Phys.- JETP* **5** (1957) 1174;
H.B. Nielsen and P. Olesen, *Vortex line models for dual strings*, *Nucl. Phys.* **B 61** (1973) 45.

[14] H. Kleinert, *Disorder version of the abelian Higgs model and the order of the superconductive phase transition*, *Lett. Nuovo Cim.* **35** (1982) 405.

[15] H. Kleinert, *Gauge fields in condensed matter. Vol. 1: Superflow and vortex lines. Disorder fields, phase transitions*, World Scientific, Singapore 1989.

[16] H. Kleinert, *Vortex, defect and monopole gauge fields for Nambu-Goldstone systems and their phase transitions or theory of fluctuating multivalued fields*, in: Proceedings of NATO Advanced Study Institute on formation and interaction of topological defects at the University of Cambridge, pp. 201–232, Plenum Press, New York 1995 [`cond-mat/9503030`].

[17] H. Kleinert, *Double gauge invariance and local quantum field theory of charges and Dirac magnetic monopoles*, *Phys. Lett.* **B 246** (1990) 127; *Abelian double gauge invariant continuous quantum field theory of electric charge confinement*, *Phys. Lett.* **B 293** (1992) 168; *The extra gauge symmetry of string deformations in electromagnetism with charges and Dirac monopoles*, *Int. J. Mod. Phys.* **A 7** (1992) 4693.

[18] P. Orland, *Extrinsic curvature dependence of Nielsen-Olesen strings*, *Nucl. Phys.* **B 428** (1994) 221 [`hep-th/9404140`].

[19] V.N. Popov, *Functional integrals in quantum field theory and statistical physics*, Reidel, Dordrecht 1983.

[20] D. Antonov, *Abelian Higgs model as an ensemble of vortex loops*, *Int. J. Mod. Phys.* **A 14** (1999) 4347 [`hep-th/9906032`]; *Ensemble of vortex loops in the abelian-projected* SU(3)*-gluodynamics*, *Mod. Phys. Lett.* **A 14** (1999) 1829 [`hep-th/9908033`]; *Field correlators in abelian-projected theories and stochastic vacuum model*, *J. High Energy Phys.* **07** (2000) 055 [`hep-ph/0006156`].

[21] G. Parisi and N. Sourlas, *Random magnetic fields, supersymmetry and negative dimensions*, *Phys. Rev. Lett.* **43** (1979) 744.

[22] A. Zamolodchikov and Al. Zamolodchikov, *Factorized S matrices in two dimensions as the exact solutions of certain relativistic quantum field models*, *Ann. Phys. (N.Y.)* **120** (1979) 253;
R. Rajaraman, *Solitons and instantons. An introduction to solitons and instantons in quantum field theory*, North-Holland Publishing Company, Amsterdam 1982;
L.D. Faddeev and L.A. Takhtajan, *Hamiltonian methods in the theory of solitons*, Springer Verlag, Berlin 1987.

[23] V.L. Berezinskii, *Description of long range order in one-dimensional and two-dimensional systems having a continuous symmetry group. 1. Classical systems*, *Sov. Phys.- JETP* **32** (1971) 493;
J.M. Kosterlitz and D.J. Thouless, *Ordering, metastability, and phase transitions in two-dimensional systems*, *J. Phys.* **C 6** (1973) 1181;
J.M. Kosterlitz, *The critical properties of the two-dimensional XY model*, *J. Phys.* **C 7** (1974) 1046;
For a review see e.g. J. Zinn-Justin, *Quantum field theory and critical phenomena*, Oxford Univ. Press Inc., 2nd Edition, New York 1993.

[24] G. Parisi, in: *Recent advances in field theory and statistical mechanics*, proceedings of the 39th Summer School, Les Houches, France, August 2- September 10, 1982, J.-B. Zuber and R. Stora eds., North-Holland Publishing Company, Amsterdam 1984.